Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 3, pp. 482–492, 2001

GENERALIZED RHEOLOGICAL MODEL OF CAVITATING CONDENSED MEDIA

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UDC 532.135:532.52;539.374

It is shown that development of cavitation in solid-plastic, liquid-plastic, and liquid media can be modeled using a rheologically equivalent, cavitating viscoelastoplastic body containing microcavities in the initial state. An energy inequality is derived that defines the loading conditions for a body with microcavities under which the body enters a cavitating state, i.e., the concentration of microcavities increases by more than an order of magnitude. A generalized rheological equation of state is formulated; analytical dependences of the modulus of volume elasticity, volumetric (second) viscosity, and the relaxation time of tensile stresses on the volume concentration of cavitational hollows in the model viscoelastoplastic body are derived.

Numerous processes studied by solid mechanics are accompanied by the growth of microcavities in the tensile-stress field (i.e., micropores in solids and microbubbles in liquids). In liquid media, this process can result in unlimited bubble growth from cavitation nuclei, transition of the medium into a foamy structure, and subsequent disintegration into fragments (cavitation damage). In the case of tensile deformation of solids, the growth of micropores and their coalescence are important components of the brittle fracture mechanism. In addition, there are intermediate media between liquid and solid materials in the rheological row: in the undisturbed state, they show an insignificant limit of shear elasticity, while under loading they enter the liquid state, losing their structural viscosity. (Among such media are gels, jellies, asphalts, concretes, paints, etc.) With allowance for the aforesaid, the cavitation development process for all these media should be described within the framework of a general model medium that possesses the properties of both rheonomous and scleronomous bodies. We assume that a cavitating medium is a medium in which the volume concentration of microcavities increases by at least an order of magnitude compared to its initial value.

The objective of the present work is to construct a generalized model of cavitating media. In terms of theoretical rheology, the media are assumed to be solid-plastic if $\tau_*/P_\infty \gg 1$ (τ_* is the yield point for the media under pure shear and P_∞ is the atmospheric pressure), liquid-plastic if $\tau_* \approx P_\infty$, and liquid if $\tau_*/P_\infty \ll 1$. Since microcavity growth in a condensed medium is due to the divergent flow of the medium in the vicinities of pores, where it is in the liquid or viscoelastic state, the medium should be subjected to volume-tensile stress such that the shear stresses in the vicinity of pores exceed the yield point. Deformation of this kind is feasible only at the stage of unloading of a medium containing microcavities after it has been shock-wave loaded.

1. To justify the possibility of constructing a generalized rheological model, we perform a comparative analysis of the behavior of cavitating media under dynamic loading and determine the features of deformation processes common for these media. For this purpose, we consider the stress–strain diagrams of condensed media, placing them in descending order of yield points τ_* from plastic metals to low-viscosity liquids.

Figure 1a shows uniaxial tension diagrams [1–3] for cylindrical samples of solid-plastic bodies [curve 1 refers to iron (α-iron), curve 2 to polycrystalline aluminum, and curve 3 to a polymer in a rigid, non-brittle state (below glass-transition temperature)]. Since these materials have incommensurable values of τ_*, Fig. 1 shows qualitative dependences $\sigma(\varepsilon)$ (σ is the tensile stress and ε is the total strain of the sample). In the case of iron, which has

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Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 3, pp. 116–129, May–June, 2001. Original article submitted June 20, 2000; revision submitted November 3, 2000.

 0021-8944/01/4203-0482 $25.00

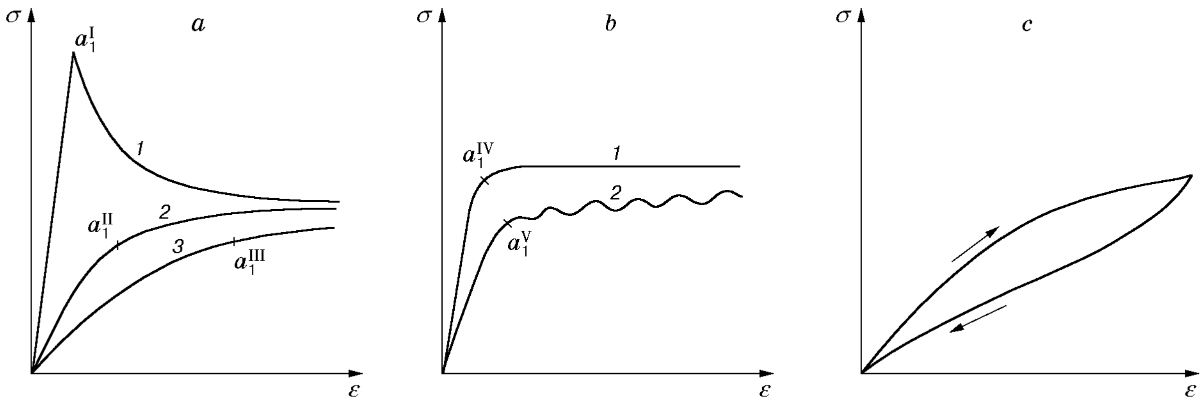

Fig. 1

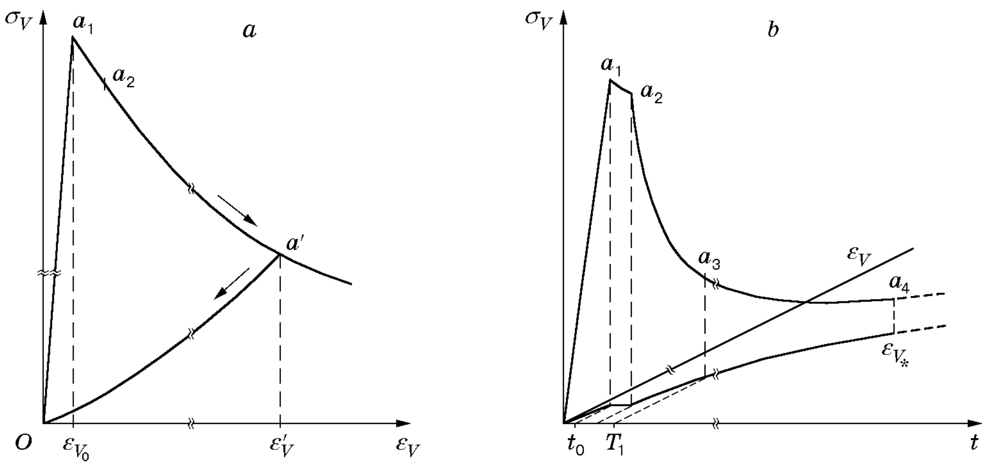

Fig. 2

the largest value of τ_* among the given materials, the diagram has a so-called "plasticity tooth." That is, when the tensile stress reaches a certain value of $a_1^{\rm I}$ (depending on the strain rate), it starts decreasing together with the continued rise in strain ε. The yield point τ_* of polycrystalline aluminum is much lower than that of iron. The dependence $\sigma(\varepsilon)$ for this material is similar to the dependence for an ideal plastic body: if $\sigma > a_1^{\rm II}$, the elastic deformation of the medium changes into yielding flow with strain increasing beyond bound at $\sigma = {\rm const}$. Polymers have qualitatively similar dependences $\sigma(\varepsilon)$ (curve 3) but lower ultimate stress $a_1^{\rm III}$.

Figure 1b gives shear strain diagrams for liquid-plastic media. Curve 1 refers to 2% gel (aluminum naphthenate solution in petrolatum) [4] and curve 2 refers to dry foam (volume concentration of hexagonal foam cells in water $\alpha \approx 0.97$) [5]. In both cases, the medium is deformed as Bingham's body, i.e., initially, elastic deformation occurs and then, after σ exceeds the threshold value ($a_1^{\rm IV}$ and $a_1^{\rm V}$, respectively), the medium losses structural viscosity and starts flowing. Figure 1c [6] shows a tensile stress-strain diagram $\sigma(\varepsilon)$ for an elastomer (linear polymer), for example, rubber. This material is capable of large (up to 1000%) reversible hyperelastic strains.

According to current concepts, Newtonian liquids do not possess shear elasticity at Deborah numbers ${\rm De} = T_0/\Delta t_* \ll 1$ (T_0 is the time of shear-stress relaxation in the medium and Δt_* is the characteristic time of stress). However, numerous experiments (see, for example, [7, 8]) give indirect evidence that liquids posses insignificant shear elasticity. Finally, Apakashev and Pavlov [9] showed experimentally that under very small strain, water behaves as a medium with an insignificant yield point and shear modulus: $\tau_* \simeq G_0 = 10^{-6}$ Pa. It was also established that for glycerin, $\tau_* \simeq G_0 = 1$ Pa at a temperature of 15°C (G_0 is the adiabatic shear modulus). Thus, homogeneous Newtonian liquids in their rheological properties can be nominally identified with Bingham's bodies, which are characterized by a very low yield point τ_*. If a liquid sample containing cavitation nuclei is subjected to volume tension, the dependence $\sigma_V(\varepsilon_V)$ takes the form shown in Fig. 2a [10], where σ_V and ε_V are the volume tensile stress and strain, respectively. If $\Delta t_* < T_0$, the increase in σ_V to the maximum value a_1 occurs in the elastic

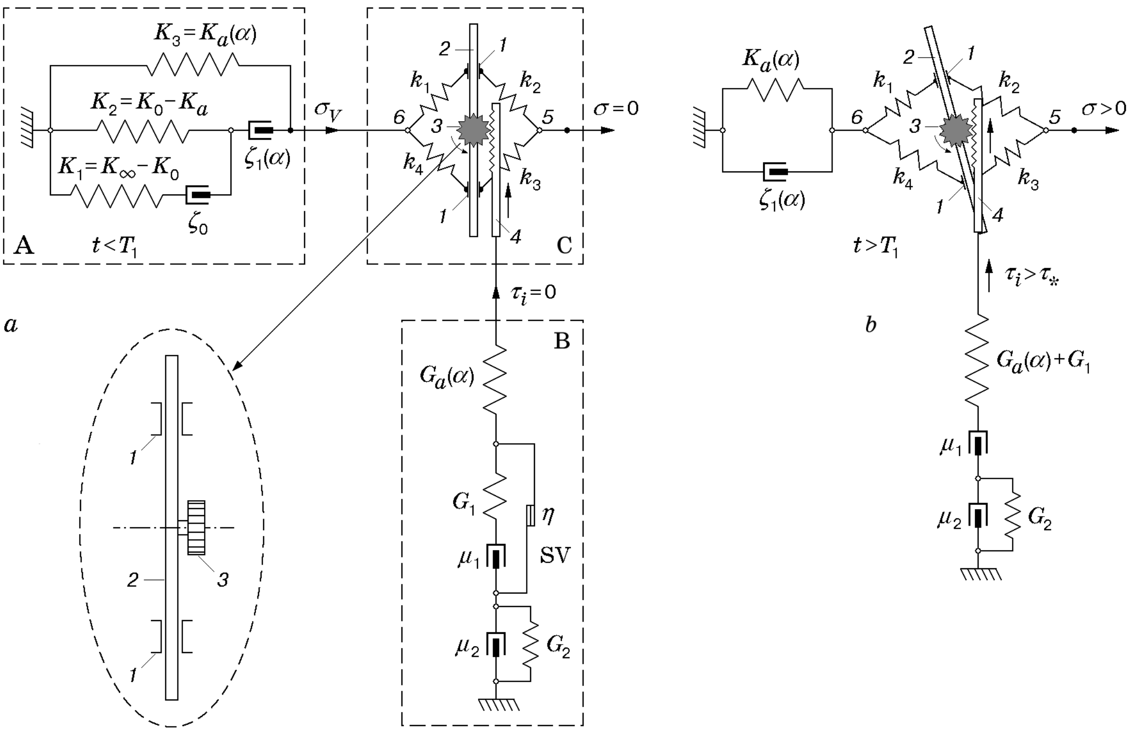

Fig. 3

regime [$T_0 = \zeta_0(K_\infty - K_0)^{-1}$, where ζ_0 is the volume viscosity and K_∞ and K_0 are the dynamic and adiabatic moduli of volume elasticity of the pure liquid, respectively]. The portion a_1a_2 of the curve in Fig. 2 corresponds to partial relaxation of σ_V due to restructuring of the liquid at the molecular level, i.e., without changes volume changes. Next, the portion a_2a_3 corresponds to the relaxation of σ_V due to unlimited bubble growth from cavitation nuclei with subsequent formation of a foamy cell structure. The next portion a_3a_4 illustrates the volume tension (in the inertial regime) of the foam up to a certain critical strain ε_{V_*} that corresponds to the onset of capillary disintegration of the medium into fragments. At this stage, the foamy medium acquires volume elasticity, which is due to atmospheric back pressure (the capability of the medium for reverting to its original state along the trajectory $a'O$ after relaxation of the stress σ_V) and the shear elasticity of the liquid films forming foamy cells [the elastic (surface) energy of the films grows with increase in ε_V]. Hence, the foamy-like medium formed during development of unlimited cavitation in a Newtonian liquid possesses both volume and shear elasticity. If the shear stresses τ exceed τ_* during deformation, the medium enters the yield state [5, 11].

From the aforesaid it follows that all solid-plastic, liquid-plastic, and liquid media possess both shear elasticity and yielding. In transition from plastic metals to low-viscosity liquids, τ_* changes by a factor of 10^{17}–10^{18}. Since all real condensed media possess shear viscosity μ, the generalized macrorheological model of cavitating media can be developed with allowance for the properties of an elastoviscoplastic body (EVPB) that contains cavitational nuclei (microcavities) in its original state. It must be considered that cavitation is a property of media that exhibit yield properties constantly or under certain loading conditions, and this allows the cavities to extend in the tensile-stress field.

2. In a linear approximation, a three-dimensional strain of a continuous medium can always be decomposed into a volumetric strain and a shear strain. Taking into account that volumetric strain is always accompanied by shear, these constituents of the mechanical model of EVPB are functionally joined with an auxiliary unit in the present work (Fig. 3). In Fig. 3a, the mechanical block A corresponds to volume tension of the body, the block B to simple shear, and the block C is an auxiliary unit that consists of a bridge comprising four spring elements $k_1, \ldots, k_4$. On one of the diagonals of the bridge there is rod 2, which is able to freely glide inside sleeves 1 (connected to the spring elements with hinges). Gear 3 is rigidly connected to the rod. When the gear is rotated (together with the rod), the mechanical block B is set in tension through rack-and-gear drive 4. The rigidity of the bridge springs satisfies the condition $k_1 = k_3 > k_2 = k_4$. Thus, if tensile force σ is applied at the junction point of the bridge 5 (Fig. 3b) there is extension of the mechanical block A (connected to the block B at point 6), which corresponds to volume deformation of the EVPB. Because of the asymmetry of the bridge, this is accompanied by extension of the block B, which corresponds to shear deformation of the EVPB.

The mechanical block A coincides with the mechanical model of volume deformation of Newtonian liquids described in [10, 13] since the qualitative character of volume deformation is identical for all condensed media [12] and can be determined only by the two rheological parameters: the modulus of volume elasticity K and the second (volumetric) viscosity ζ, which generally depend on the volume concentration of cavitational hollows α. The block A consists of elastic elements $K_1 = K_\infty - K_0$, $K_2 = K_0 - K_a$, and $K_3 = K_a(\alpha)$ (K_a is the modulus of volume elasticity of a body containing cavitational hollows of concentration α) and viscous elements ζ_0 and $\zeta_1(\alpha)$, which are the volume viscosity of the homogeneous EVPB and the volume viscosity of the EVPB containing cavitational hollows, respectively.

The block B consists of a series of mechanical units corresponding to Shvedov's and Kelvin–Voigt's bodies. The first unit comprises the elastic element $G_a(\alpha)$, which corresponds to the shear elasticity of the EVPB, the elastic element G_1, and the piston μ_1, which corresponds to the shear viscosity of the medium, so that

$$\mu_1 = \begin{cases} \mu_0 & \text{for} \quad \alpha = 0, \\ \mu_* & \text{for} \quad 0 < \alpha \leqslant \alpha_*, \\ \mu_{**} & \text{for} \quad \alpha > \alpha_*. \end{cases}$$

Here $\alpha_* \simeq 0.74$–0.76 is the volume concentration of cavities for their limiting packing (feasible only in liquid media), μ_0 is the shear viscosity of the homogeneous medium, μ_* is the effective shear viscosity of the gaseous suspension (medium containing cavitational hollows), and μ_{**} is the structural viscosity of a foamy cell system, feasible only for unlimited cavitation of low-viscosity liquids or liquid-disperse media and gels having a low-viscosity liquid as a matrix [14, 15]. The Saint-Venant (SV) element is connected in parallel to the elements μ_1 and G_1 and blocks them in various modes. The SV element corresponds to the plastic viscosity of the medium η, which varies as

$$\eta(\tau, \alpha) = \begin{cases} \infty & \text{for} \quad \tau < \tau_*, & \alpha < \alpha_*, \\ \eta_* & \text{for} \quad \tau = \tau_*, & \alpha < \alpha_*, \\ 0 & \text{for} \quad \tau > \tau_*, & \alpha < \alpha_*, \\ \infty & \text{for} \quad \tau_* < \tau < \tau_{**}, & \alpha > \alpha_*, \\ \eta_{**} & \text{for} \quad \tau = \tau_{**}, & \alpha > \alpha_*, \\ 0 & \text{for} \quad \tau > \tau_{**}, & \alpha > \alpha_*. \end{cases}$$

Here τ_* is the yield point of the homogeneous EVPB and τ_{**} is the yield point of the foamy cellular structure into which the EVPB evolves at $\alpha > \alpha_*$. As $G_1 \to 0$, the mechanical unit simulating Shvedov's body (for example, plastic gels) degenerates into the unit corresponding to Bingham's body. At $\alpha < \alpha_*$, the Kelvin–Voigt unit $\mu_2|G_2$ connected in series with the mechanical model of a Shvedov's body corresponds to the viscoelastic properties of cavitational hollows, and at $\alpha > \alpha_*$ it corresponds to the properties of foamy cells. For polydisperse cavitational hollows, the generalized Kelvin–Voigt units includes the entire spectrum of elements $\mu_2^i|G_2^i$ corresponding to the spectrum of typical cavity sizes $\{d_i\}$.

The block B operates as follows. Tensile force σ applied to the point 5 of the block B gives rise to "volume" deformation of the block A due to the applied tensile force σ_V and "shear" deformation of the block B due to the force τ (corresponding to pure shear stress). In this case, the mode of operation of the block B depends on the value of α and the ratio between τ and τ_*. The latter can take different values from 10^{-6} Pa for water, 1 Pa for glycerin, and up to 10^{10} Pa for steel.

If $\tau < \tau_*$, then $\eta = \infty$, i.e., the SV element is closed, thus blocking the piston μ_1 with the spring G_1, the medium behaves as an elastic solid body containing viscoelastic disperse elements (pores), which are equivalent to the generalized Kelvin–Voigt body $\mu_2|G_2$. Under these loading conditions, the medium is either a scleronomous gaseous suspension at $\alpha < \alpha_*$ or a cellular structure with a solid elastic matrix at $\alpha > \alpha_*$.

If $\tau > \tau_*$, then $\eta = 0$, i.e, the SV element is opened, the piston μ_1 starts moving, and the medium acquires viscoplastic properties. (If $G_1 \neq 0$, this process at a relevant strain rate is accompanied by accumulation of elastic

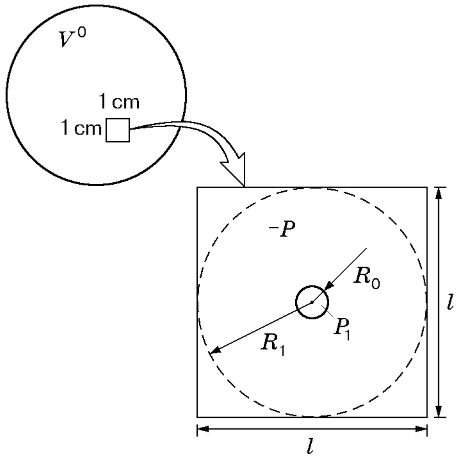

Fig. 4

energy in the element G_1.) In this case, if $\alpha \to 0$, we have a classical Maxwell body, whose behavior depends on the relation of the parameters $G_a + G_1$ and μ_0, and the Deborah number $\mathrm{De} = \mu_0[(G_a + G_1)\Delta t_*]^{-1}$. When the values of μ_0 are very large, we have solid-plastic bodies. If $\mu_0 \leqslant 10^2$ Pa · sec, we have liquids [Newtonian liquids at $\mu_0 = \mathrm{const}$ and non-Newtonian ones at $\mu_0 = \mu_0(\dot{\varepsilon}_\tau)$, where $\dot{\varepsilon}_\tau$ is the shear strain rate]. In the last case at $\mathrm{De} \gg 1$, the piston μ_1 has no time to move; therefore, the shear strain is determined only by the elastic element $G_a + G_1$, i.e., the liquid behaves as Hooke's body. At $\mathrm{De} \leqslant 1$, the behavior of the medium depends on the shear viscosity μ_1 and corresponds to a plastic or liquid body.

With increase in α, the medium acquires the properties of a gaseous suspension, while at $\alpha > \alpha_*$ (which is possible only in the case of liquid low-viscosity matrices [14–16]), the bubbles are consolidating and the medium is transformed to a rigid foamy framework. In this case, if $\tau < \tau_{**}$, then $\eta = \infty$, i.e., the SV element is closed and the medium has the properties of a viscoelastic body. If $\tau > \tau_{**}$, the SV element opens ($\eta = 0$), and in the medium there is plastic deformation of the cellular structure. The plastic deformation properties depend on the capillary number Ca [11]. We note that all the aforesaid about gaseous suspensions with a liquid matrix and foamy structures is also valid for foamed metals and other scleronomous media at both $\alpha < \alpha_*$ and $\alpha > \alpha_*$ (if this structure is feasible). However, since the present paper deals only with development of cavitation in condensed media, we further consider the cases where $\alpha \ll \alpha_*$ and the volume average tensile stress in the medium is located in a zone bounded by the yield surface.

Thus, the analysis performed demonstrates that the constructed generalized mechanical model of an EVPB (see Fig. 3) adequately describes all principal materials capable of cavitating.

3. Let us analyze conditions for the growth of cavitational hollows in the modeled EVPB. We assume that the modeled undisturbed EVPB with density ρ_0 contains monodisperse microcavities of initial radius R_0 with number density n, uniformly distributed over volume V^0 (Fig. 4). Then, by analogy with [13], we split the unit volume of the medium into n cubic cells, so that the center of each cell contains a spherical cavity of radius R_0 and the cell size is $l \times l \times l$ ($l = n^{-1/3}$). The surface tension on the boundary of the cavity is γ; generally, P_1 is the pressure inside the cavity (if the medium is a liquid, $P_1 = P_\mathrm{v} + P_\mathrm{g}$, where P_v is the vapor pressure and P_g is the gas pressure), and R_0 and n satisfy the condition $R_0 \ll n^{-1/3}$.

From analysis of the growth conditions obtained in [13, 16] for pores in scleronomous media and bubbles in liquids, it follows that the growth of cavitational hollows in condensed media depends on their strength properties, i.e., parameter τ_*, viscosity μ_1, surface tension γ, and atmospheric back pressure P_∞. The parameter τ_* is dominant for liquid-plastic media, μ_1, for high-viscosity liquids and scleronomous media deformed in the plastic yield regime, and γ and P_∞, for low-viscosity liquids. From the results obtained in [13], it follows that if a negative pressure $P < 0$ is applied to an EVPB containing microcavities of initial radius R_0 (Fig. 4), so that the Tresca plasticity condition, expressed in this case by the inequality

$$\tau = \frac{3|\hat{P}|}{4\bar{r}^3} > \tau_* \qquad (1)$$

is valid in the vicinity of a microcavity, the dynamic equation takes the form

$$b\ddot{b} + \frac{3-4\beta_0}{2(1-\beta_0)}\,\dot{b}^2 + h\,\frac{\dot{b}}{b} = Qh(1-s). \tag{2}$$

Here $b = RR_0^{-1}$ (R_0 and R are the initial and current radii, respectively),

$$h = \frac{4\mu_1}{\rho R_0^2(1-\beta_0)}, \quad Q = \frac{\hat{P}}{4\mu_1}, \quad s = D\Big[\ln\beta^{-1} + \frac{1}{3}\Big(1 - \frac{a^3}{R_c^3}\Big)\Big] = D\Big(\ln\beta^{-1} + \frac{1}{3}\,(1-\alpha\beta^{-3})\Big),$$

$$D = 4\tau_*|\hat{P}|^{-1} \simeq 2Y_*|\hat{P}|^{-1}, \qquad \beta = \frac{R}{a} = \Big[4\tau_*(3|\hat{P}|\,)^{-1}\Big]^{1/3}\Big|_{\hat{P}=\mathrm{const}} = \beta_0 = \frac{R_0}{a_0}, \tag{3}$$

a is the radius of the plasticity zone in the vicinity of a microcavity, where condition (1) is satisfied, Y_* is the tensile yield point, $R^3R_c^{-3} = \alpha$ ($R_c = 0.5l = 0.5n^{-1/3}$ is the radius of the circle inscribed to the cell), and $\hat{P} = -P + P_1 - P_\infty - 2\gamma R_0 b^{-1}$ ($P < 0$ for extension and $P > 0$ for compression).

It follows from (2) that the boundary of the cavity is in equilibrium ($\dot{b} = \ddot{b} = 0$) if the condition $Qh(1-s) = 0$ is satisfied, which, with allowance for (3), can be written as $\hat{P} - 4\tau_*(\ln\beta^{-3} + 1 - \alpha\beta^{-3})/3 = 0$. To make this condition valid for the collapse (compression) regime of a cavity, we write it as

$$\hat{P} = (-1)^{m+1}4\tau_*(\ln\beta^{-3} + 1 - \alpha\beta^{-3})/3, \tag{4}$$

where $m = 1$ corresponds to extension of the medium and $m = 2$ to compression. If $\hat{P} = -P$, this condition agrees with the well-known Hencky's formula from solid-state mechanics, which defines the equilibrium condition for a pore wall in a spherical elastoplastic body. According to (4), when the plasticity zone is beginning to form on the cavity boundary and $a = R$, the pressure corresponding to the plastic threshold has the form

$$\hat{P}^0 = (-1)^{m+1}4\tau_*(1-\alpha)/3. \tag{5}$$

According to (4), when $a \simeq R_c = 0.5n^{-1/3}$, the pressure corresponding to the moment when the outer radius of the plastic layer a reaches the cell boundary is given by

$$\hat{P}^* = (-1)^{m+1}4\tau_*\,\ln(\alpha^{-1})/3. \tag{6}$$

It is worth noting that for $\hat{P} \to P$, the problem of pore dynamics in solid materials considered in [17] is relevant. At $\tau_* \to 0$, Eq. (2) reduces to the equation of motion for a spherical bubble wall in a viscous liquid and Eq. (4) reduces to the equilibrium condition for a bubble wall [18]. Equations (5) and (6) at $\hat{P} = P$ derived from Hencky's formula were used in papers [19–21], dealing with mathematical modeling of dynamic processes in porous materials. Kiselev [19] derived an analytical dependence for the modulus of volume elasticity of a material on porosity and loading pressure, which is of interest for the present work.

According to the experimental data of [18, 22], the initial volume concentration of microcavitites α_0 in liquid and solid-plastic media lies in the range of 10^{-12}–10^{-7}. Since the problem under study concerns cavitation development from initial microcavities, i.e., the growth of cavities to values equal to at least $\alpha \simeq 10\alpha_0$, we can restrict ourselves to cavity radii of $R < 10R_0$ and, hence, the condition $a^3/R_c^3 \ll 1$. With allowance for this, when deriving the conditions of growth to values of $b_* \simeq 10$, it is assumed that $s = D(\ln\beta^{-1} + 1/3) \simeq 4\tau_*(\ln\beta^{-3} + 1)/3$.

To determine the range of negative pressures in which the medium is cavitating (i.e., $\alpha_{\max} \geqslant 10\alpha_0$ and $b \geqslant b_* = \sqrt[3]{10}$), the condition for cavity growth in an EVPB to specified size was derived by analogy with the condition for pore growth in solid-plastic media [13]:

$$\Big(\frac{1}{3K} + \frac{1}{G}\Big)\tilde{P}^2\Big[\frac{b_*^3}{\beta_0^3} - 1 + \frac{(1+\xi)^2}{2}\Big(1 - \frac{\beta_0^3}{b_*^3}\Big)\Big] > 4\tau_*(b_*^3 - 1)\Big(\ln\beta_0^{-1} + \frac{1}{3}\Big)$$

$$+ 2P_\infty(b_*^3 - 1) + \frac{6\gamma}{R_0}\,(b_*^2 - 1) + 3\rho R_0^2[(1-\beta_0)b_*^3\dot{b}_*^2 - \beta_0 J_1] + 24\mu_1 J_2. \tag{7}$$

Here $K = K_a$, $G = G_a$, $\tilde{P} = |P| - P_\infty$, $\xi = [P_1 - 2\gamma(R_0 b)^{-1}]\tilde{P}^{-1}$, $J_1 = \int\limits_0^{t_*} b\dot{b}^3\,dt$, and $J_2 = \int\limits_0^{t_*} b\dot{b}^2\,dt$. For solid-plastic media [13], we have

$$J_1 = Mh^2\Big\{[\hat{t}_0 + 5\exp(-\hat{t}_0)]M + \frac{1}{5}\,[\exp(5\,\ln\,b_*) - 1]\Big\},$$

$$J_2 = M^2h\Big[\hat{t}_0 + 2\exp(-\hat{t}_0) + \frac{1}{3}\,\exp(3\,\ln\,b_*) - \frac{11}{6} - \frac{1}{2}\exp(-2\hat{t}_0)\Big],$$

where $M = Q(1-s)^{-1}$, t is the time elapsed from the moment of negative-pressure loading of the cell, $\hat{t} = ht$, and $\hat{t}_0$ is determined from the equation

$$(3-4\beta_0)\exp(-\hat{t}_0) + (1-2\beta_0)\exp(-\hat{t}_0) - 2(1-\beta_0)\hat{t}_0 = 2(1-\beta_0)M^{-1} + 2(2-3\beta_0). \qquad (8)$$

The generalized inequality (7), unlike the conditions for pore growth in solid-plastic media [13], takes into account the effect of the parameters P_∞ and γ on cavity growth, which is essential for the development of bubble cavitation in liquids. Thus, if the store of elastic energy in a cell due to the negative pressure applied [the left side of inequality (7)] exceeds the work on extending the cavity radius to $R_* = \sqrt[3]{10}$ [the right side of inequality (7)], the medium is considered cavitating at the given loading level.

In solid-plastic media, most of the elastic energy stored in a cell under volume tension is expended in overcoming structural viscosity (strength forces), i.e., in transition to a plastic state, as is shown in [13]. Hence, all terms on the right side of (7), except for the first one, can be neglected. Because the left side of this inequality contains $b_*^3\beta_0^{-3} \gg 0.5(1+\xi)^2(1-\beta_0^3 b_*^{-3}) - 1$, it can be reduced to

$$\Omega b_*^3\beta_0^{-3} > 1.5s(b_*^3 - 1),$$

where $\Omega = \tilde{P}E^{-1}$ and $E = KG(G+3K)^{-1}$ is Young's modulus.

In transition to liquid media, τ_* assumes near-zero values for water, for example, $\tau_* \simeq G = 10^{-6}$ Pa. Therefore, since $\dot{b}_* = 0$ (as $b_* = \max\{b\}$ by definition) and $K \gg G$ and $\beta_0^3 \ll 1$ for liquids, condition (7) is brought to the form

$$\frac{\tilde{P}^2}{2G}\Big[\frac{b_*^3}{\beta_0^3} - 1 + \frac{(1+\xi)^2}{2}\Big] + \frac{3}{2}\,\rho R_0^2\beta_0 J_1 > P_\infty(b_*^3 - 1) + \frac{3\gamma}{R_0}\,(b_*^2 - 1) + 12\mu_1\int_0^{t_*} b\dot{b}^2\,dt. \qquad (9)$$

The second term on the left side of inequality (9) is always much smaller than the first term, and, therefore, it can be neglected. The right side of inequality (9) coincides with the right side of the inequality obtained in [16], which determines the conditions for bubble growth in liquids. The left sides of these inequalities differ because the condition formulated in [16] is valid for bubble growth under relaxing negative pressure, whereas the generalized condition for bubble growth (9) is valid for the cases where negative constant pressure is applied to a cell, which is justified for the problem formulated in the present paper (investigation of the conditions for cavity growth to the size $R = \sqrt[3]{10}\,R_0$ in condensed media).

4. Using the generalized mechanical model constructed (see Fig. 3), we formulate the rheological equation of volume tension of the modeled EVPB. Since the same mechanical model describes various volume deformations of all condensed media (block A in Fig. 3a), the general form of the rheological equation that refers to the volume deformation of the modeled EVPB coincides with the rheological equation for liquids under volume tension [10]

$$\ddot{\sigma}_V + \Big(\frac{1}{T_0} + \frac{Z}{T_1}\Big)\dot{\sigma}_V + \frac{\sigma_V}{T_0T_1} = K_\infty\ddot{\varepsilon}_V + \Big(\frac{K_0}{T_0} + \frac{ZK_0}{T_1}\Big)\dot{\varepsilon}_V + \frac{K_a}{T_0T_1}\,\varepsilon_V. \qquad (10)$$

Here $Z = (K_\infty - K_a)(K_0 - K_a)^{-1}$, $T_0 = \zeta_0(K_\infty - K_0)^{-1}$, and $T_1 = \zeta_1(K_\infty - K_a)^{-1}$.

Let us derive the rheological functions $K_a(\alpha)$ and $\zeta_1(\alpha)$ for two loading ranges: $|\hat{P}| < |\hat{P}^0|$ and $|\hat{P}^0| < |\hat{P}| < |\hat{P}^*|$. Here $\hat{P}^0$ is derived from (5).

Elastic Deformation in the Range of Negative Pressures $|\hat{P}| < |\hat{P}^0|$. In a cell we distinguish a spherical layer whose internal boundary is a cavity of radius R_0 and whose external boundary is a sphere of radius $R_c = 0.5\sqrt[3]{n}$ inscribed in the cubic cell (see Fig. 4). We assume that the pressure P is applied to the external boundary of the layer. Then, placing the origin of spherical coordinates (r, θ, φ) at the center of the cavity and using the solution of the well-known elastic problem [23], we obtain the following expressions for the elastic-stress tensor components:

$$\varepsilon_{rr} = \tilde{A} - 2Br^{-3}, \qquad \varepsilon_{\theta\theta} = \varepsilon_{\varphi\varphi} = \tilde{A} + Br^{-3}. \qquad (11)$$

With allowance for this, the formula of volume strain becomes

$$\varepsilon_V = \varepsilon_{ii} = \varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{\varphi\varphi} = 3\tilde{A}. \qquad (12)$$

To determine $\tilde{A}$ from (12) and the Hooke's generalized law $\varepsilon_{ik} = (9K)^{-1}\delta_{ik}\sigma_{ll} + (2G)^{-1}(\sigma_{ik} - 0.333\delta_{ik}\sigma_{ll})$, we have $\sigma_{rr} = 3K_0\tilde{A} - 4G_0Br^{-3}$, from which, taking into account the boundary conditions $\sigma_{rr} = -P_1$ at $r = R_0$ and $\sigma_{rr} = -P$ at $r = R_c$, the expressions for accurately determining the cell volume $V_c = (2R_c)^3$, and the inequality $\alpha|P_1| \ll |P|$, we obtain

$$\tilde{A} = (P_1R_0^3 - PR_c^3)[3K_0(R_c^3 - R_0^3)]^{-1} \simeq -P[3K_0(1 - 6\alpha/\pi)]^{-1}. \tag{13}$$

Finally, substituting (13) into (12), we have

$$\varepsilon_V = -P[K_0(1 - 6\alpha/\pi)]^{-1} \qquad (P < 0). \tag{14}$$

Since in terms of linear viscoelasticity, the three-dimensional governing equation for volume changes has the form $\sigma_{ii} = 3K\varepsilon_{ii} = 3K\varepsilon_V$, where $\sigma_{ii} = -3P$, we can write $K = -P\varepsilon_V^{-1}$ or, for a medium containing cavitational hollows of concentration α,

$$K_a(\alpha) = -P[\varepsilon_V(\alpha)]^{-1}, \qquad P < 0. \tag{15}$$

Then, substituting (14) into (15), for the modulus of volume elasticity of the medium loaded by pressures at which this medium is deformed as an elastic body, we obtain

$$K_a(\alpha) = (1 - 6\alpha/\pi)K_0, \qquad K_0 = K_a \qquad (\alpha = 0), \qquad 0 \leqslant \alpha < \alpha^0. \tag{16}$$

Here α^0 is the volume concentration of cavitational hollows that corresponds to the maximum allowable radii R^0 satisfying the condition $R^0 \ll 0.5l = 0.5n^{-1/3}$, which is necessary for the existence of solution (11).

Viscoelastoplastic Deformation in the Pressure Range $|\hat{P}^0| < |\hat{P}| < |\hat{P}^*|$. In this case, a field of shear stresses τ exceeding the yield point τ_* forms in the vicinity of each cavitational hollow during deformation, and, hence, divergent plastic flow develops, i.e., the cavities are expanded. Since the modulus of volume elasticity of the medium is a quantitative characteristic of reversible volume changes, i.e., deviations from the equilibrium value under the pressure applied, determination of the modulus makes sense only for the medium in the equilibrium state. We assume that the dependence of the gas pressure in a cavitational hollow on its radius has the general form $P'_{\rm g} = P_{\rm g}z^{-3\bar{k}}$, where $P_{\rm g}$ is the pressure in the cavity in the equilibrium state, $z = R'/R$, and $\bar{k}$ is the polytropic exponent (unprimed variables correspond to the equilibrium state of the system and variables with primes refer to the disturbed state). For liquid media with bubbles, $P_{\rm v}$ is always equal to the saturated vapor pressure, i.e., $P'_{\rm v} = P_{\rm v} = \text{const}$, and for solid-plastic materials with pores, $P_{\rm v} = 0$. Then, substituting the expression $P_{\rm g} = P - P_{\rm v} + 2\gamma/R + (-1)^{m+1}4\tau_*[\ln{(a/R)^3} + 1 - \alpha(a/R)^3]/3 = A$ from (4) into the equilibrium equation for the cavity wall, for the disturbed pressure P', we obtain

$$-P'\Big|_{\alpha'\simeq\alpha} = (-1)^{m+1}4\tau_*[\ln{(a'/R')^3} + 1 - \alpha(a'/R')^3]/3 + P_\infty - P_{\rm v} + 2\gamma/R' - Az^{-3\bar{k}}. \tag{17}$$

For extension, $P' < 0$, and for compression, $P' > 0$. However, the first term of the right side of (17) should have the same sign as $-P'$ because for solid-plastic media, the sign on the right side of the equation is determined by the first term, whose absolute value is much larger than that of the other terms. With allowance for this, Eq. (17) is written as

$$P'\Big|_{\alpha'\simeq\alpha} = \frac{4}{3}\,\tau_*[\ln{(\beta^{-3}z^{-3})} + 1 - \alpha\beta^{-3}z^{-3}] + (-1)^{m+1}\Big(P_\infty - P_{\rm v} + \frac{2\gamma}{Rz} - Az^{-3\bar{k}}\Big). \tag{18}$$

Here we always have $P' > 0$ and loading characteristics are allowed for by the factor $(-1)^{m+1}$ ($m = 1$ for extension and $m = 2$ for compression), and, hence, (18) is equivalent to Eq. (17). From (18) we have

$$\frac{dP'}{dz}\Big|_{z=1} = -\Big[4\tau_*(1 - \alpha\beta^{-3}) - (-1)^m\Big(\frac{2\gamma}{R} - 3\bar{k}A\Big)\Big]. \tag{19}$$

For the initial equilibrium state, the formula for the volume of a condensed medium containing cavities has the form $V^0 = V_0 + V_1 = V_0 + 4\pi R^3N/3$, where V_0 is the volume of the condensed component ignoring the volume of the cavities V_1 and N is the number of cavities in the volume V^0, which satisfies the condition $\alpha < \alpha_*$. For the state corresponding to $R = R'$, this formula becomes $V^{0\prime} = V_0 + V'_1 = V_0 + 4\pi(R')^3N/3$. From the last two expressions, we obtain

$$dV^0 = dV_0 + dV_1 = dV_0 + 12\pi R^2N\,dR/3 = dV_0 + 3V_1\,dz. \tag{20}$$

Using the expression for the modulus of volume elasticity $K = -V\,dP/dV$, with allowance for (19) and (20), and the fact that $\alpha = V_1/V^0$ and $V_0/V^0 = (V^0 - V_1)/V^0 = 1 - \alpha$, we have

$$K_a = -\frac{V^0\,dP}{dV^0} = -\frac{V^0\,dP}{dV_0 + 3V_1\,dz} = -\Big[(1-\alpha)(V_0)^{-1}\frac{dV_0}{dP} + 3\alpha\,\frac{dz}{dP}\Big]^{-1}\Big|_{z=1}$$

$$= \Big[(1-\alpha)K_0^{-1} + \frac{3\alpha}{4\tau_*(1-\alpha\beta^{-3}) - (-1)^m(2\gamma/R) - 3\bar{k}A}\Big]^{-1}. \qquad (21)$$

For a bubbly liquid, with allowance for the smallness of τ_* and assuming that gas expansion in a growing bubble is an isothermal process (by virtue of the large heat capacity of liquids), i.e., assuming that $\bar{k} = 1$, Eq. (21) reduces to the formula $K_a = \{(1-\alpha)K_0^{-1} + (-1)^m 3\alpha/[3(P + P_\infty - P_{\rm v}) + 4\gamma/R]\}^{-1}$ derived in [10]. We note that in liquids, bubbles can be in equilibrium only at $P > 0$, and, hence, K_a can be determined only at $m = 2$. Because in solid-plastic media, pore growth to concentrations $\alpha > \alpha_*$ is impossible, the dependence $K_a(\alpha)$ in the range of $\alpha_* \leqslant \alpha < 1$ is valid only for cavitating liquids. This dependence was derived in [10]. With allowance for this and formulas (16) and (21), the dependence of the modulus of volume elasticity of an EVPB on the cavity concentration is written as

$$K_a = \begin{cases} (1 - 6\alpha/\pi)K_0 & \text{for } 0 < |\hat{P}| < |\hat{P}^0|, \quad 0 \leqslant \alpha < \alpha^0, \\ \Big[(1-\alpha)K_0^{-1} + \dfrac{3\alpha}{4\tau_*(1-\alpha\beta^{-3}) - (-1)^m(2\gamma R^{-1} - 3\bar{k}A)}\Big]^{-1} & \\ & \text{for } |\hat{P}^0| < |\hat{P}| < |\hat{P}^*|, \quad 0 \leqslant \alpha < \alpha_*, \\ \tilde{\rho}^0 \tilde{C}_0^2 - (2\gamma/R_0)\sqrt[3]{\sqrt{2}\alpha_0(1-\alpha)\alpha^2/(9\pi)} & \text{for } |\hat{P}^*| < |\hat{P}|, \quad \alpha_* \leqslant \alpha < 1. \end{cases} \qquad (22)$$

Here $\tilde{\rho}^0$ is the unperturbed density of the vapor-gas fill of a foamy cell and $\tilde{C}_0$ is the speed of sound in the vapor–gas fill.

The values of $K_a(\alpha)$ calculated for liquid media agree well with experimental data (see [10]). In the case of solid-plastic media, the calculations by formula (22) [with allowance for $4\tau_*(1-\alpha\beta^{-3}) \gg |2\gamma R^{-1}|$ and $\bar{k} = 0$] were compared with similar calculations by the formula for K_a of solid porous bodies derived in [19] by a different method. The results obtained show the following: if we introduce the notation $\omega = K_{a1}/K_{a2}$, where K_{a1} is calculated by formula (22) and K_{a2}, by formula (28) from [19], for a copper sample with micropores under loading by negative pressures $0 < |\hat{P}| < |\hat{P}^0|$, we obtain $\omega = 1$ at $\alpha = 10^{-8}$, $\omega = 1.000\,02$ at $\alpha = 10^{-5}$, $\omega = 1.0019$ at $\alpha = 10^{-3}$, and $\omega = 1.019$ at $\alpha = 10^{-2}$. When the sample is loaded by negative pressures $|\hat{P}^0| < |\hat{P}| < |\hat{P}^*|$, we have the following:

— for $\chi = P/Y_* = 0.7$, $\omega = 0.999\,97$ at $\alpha = 10^{-8}$, $\omega = 0.9972$ at $\alpha = 10^{-6}$, $\omega = 0.9722$ at $\alpha = 10^{-5}$, and $\omega = 0.78$ at $\alpha = 10^{-4}$;

— for $\chi = 4$, $\omega = 0.999\,97$ at $\alpha = 10^{-8}$, $\omega = 0.998$ at $\alpha = 10^{-6}$, $\omega = 0.973$ at $\alpha = 10^{-5}$, and $\omega = 0.785$ at $\alpha = 10^{-4}$;

— for $\chi = 7.5$, $\omega = 1$ at $\alpha = 10^{-8}$, $\omega = 1.002$ at $\alpha = 10^{-6}$, $\omega = 1.011$ at $\alpha = 10^{-5}$, and $\omega = 1.23$ at $\alpha = 10^{-4}$.

Thus, the difference in calculated values of K_a is significant only when the initial pore concentration is high. For example, for a copper sample with $\chi = 4$ and $\alpha = 10^{-3}$, formula (22) yields $K_a = 0.28 \cdot 10^{11}$ Pa, and formula (28) derived in [19] yields $K_a = 1.09 \cdot 10^{11}$ Pa. As is noted above, the initial concentration of micropores in solid-plastic nonporous materials does not exceed values of $\alpha_0 = 10^{-8}$–10^{-6}. However, the goal of the present work is to examine the conditions for cavitation growth in condensed media where α increases by an order of magnitude, i.e., does not exceed the value of 10^{-5}.

It is known that all condensed media behave the same under volume extension: volume deformation gives rise to viscous strength, which is characterized by volumetric (second) viscosity [12]. Therefore, the procedure of constructing the dependence $\zeta_1(\alpha)$ for bubble liquid media described in detail in [16] is valid for any condensed media including the modeled EVPB with cavitational microhollows. With allowance for this, the rate of energy dissipation in a homogeneous medium that is rheologically equivalent to the medium studied (and has the same volume V^0) is written as

$$D = V^0 \zeta_1 \dot{\varepsilon}_V^2 = D_0 + D_b, \qquad \dot{\varepsilon}_V = \dot{\varepsilon}_{V_0} + \dot{\varepsilon}_{V_b}, \qquad (23)$$

where $D_0 = V_0 \zeta_0 \dot{\varepsilon}_{V_0}^2$ is the energy-dissipation rate in the volume V_0 of the medium at $\alpha = 0$ and $D_b = 16\pi\mu_0 R\dot{R}^2 N$ is the rate of energy dissipation due to growth of cavitational hollows in the medium under study, i.e., in the volume V^0. Assuming that $K_a(\alpha_0) \simeq K_0$ and following [16], we have

$$\dot{\varepsilon}_{V_0} = \dot{\sigma}_{V_0} K_0^{-1}, \qquad \dot{\varepsilon}_{V_b} = 3\alpha \dot{R} R^{-1}. \tag{24}$$

Substituting (24) into (23) and rearranging the equation, we finally obtain

$$\zeta_1(\alpha) = \begin{cases} \zeta_0(1-\alpha)^{-1} & \text{for} \quad 0 < |\hat{P}| < |\hat{P}^0|, \\ \zeta_0(1-\alpha)^{-1}[1 + 3\alpha \dot{R} R^{-1} K_0 \dot{\sigma}_{V_0}^{-1}]^{-2} & \\ \quad + 12\mu_0 \alpha \dot{R}^2 R^{-2} (\dot{\sigma}_{V_0} K_0^{-1} + 3\alpha \dot{R} R^{-1})^{-2} & \text{for} \quad |\hat{P}^0| < |\hat{P}| < |\hat{P}^*|. \end{cases} \tag{25}$$

From (25) it follows that for elastic deformation and plastic yield at an early stage of extension or under instantaneous loading, where De $\gg 1$ and $\dot{\sigma}_{V_0} R^{-1} \gg 1$, the volume viscosities coincide and are equal to $\zeta_1 = \zeta_0(1-\alpha)^{-1}$. In the subsequent stages of cavity growth, $3\alpha K_0 \dot{R} \gg R\dot{\sigma}_{V_0}$, $\dot{\sigma}_{V_0} K_0^{-1} \ll 3\alpha \dot{R} R^{-1}$, and $\zeta_1 \to 4\mu_0 \alpha^{-1}$, i.e., to the value of $\zeta_1(\alpha)$ in concentrated suspensions [16]. Since $\dot{\sigma}_{V_0} = -\dot{P}$, $\zeta_1(\alpha)$ generally depends on the variation of P with time. For example, if the loading dynamics has the form $P(t) = P(0)\exp(-t/t^*)$, where t^* is the time constant, and $P(t)$ "ensures" satisfaction of the elasticity condition over fixed time, (25) reduces to the dependence (18) from [16].

Thus, the constructed rheological functions $K_a(\alpha)$ and $\zeta_1(\alpha)$ completely defines the rheological equation of an EVPB with cavitational hollows (10) for all modeled media.

5. The time of relaxation of volume tensile stresses and, hence, the time of relaxation of negative pressures acting on a condensed medium are defined by the relation [12] $T = \zeta K^{-1}$. Substituting the values of K_a and ζ_1 from (22) and (25), respectively, into this relation, we obtain

$$T(\alpha) = \begin{cases} T(\alpha)[(1-\alpha)(1-6\alpha/\pi)]^{-1} & \text{for} \quad 0 < |\hat{P}| < |\hat{P}^0|, \quad \alpha < \alpha^0, \\ \Big[\dfrac{\zeta_0}{(1-\alpha)(1 + 3\alpha K_0 \dot{b}/(b\dot{\sigma}_{V_b}))^2} + \dfrac{12\mu\alpha}{(3\alpha + b\dot{\sigma}_{V_0}/(\dot{b}K_0))^2}\Big] [(1-\alpha)K_0^{-1} + (-1)^m 3\alpha P_f^{-1}] & \\ & \text{for} \quad |\hat{P}^*| < |\hat{P}|, \qquad 0 \leqslant \alpha < \alpha_*, \end{cases} \tag{26}$$

where $P_f = 3(P + P_\infty - P_\mathrm{v}) + 4\gamma/R$.

Relation (26) is valid for estimating the time of relaxation negative pressures at which the entire cell containing a microcavity is in one of the two possible homogeneous rheological states: elastic state or viscoplastic state. If $|\hat{P}^0| < |\hat{P}| < |\hat{P}^*|$ and $a < 0.5n^{-1/3}$, then the rheological state of the cell is inhomogeneous: in the layer $R \leqslant r \leqslant a$, the medium is in the viscoplastic state, and in the layer $a < r \leqslant 0.5n^{-1/3}$, it is in the elastic state. In this case, relation (26) is inapplicable, and $T(\alpha)$ is determined from the relation derived in [13]:

$$T(\alpha) = t_0 + \frac{1}{3Q(1-s)} \ln \left[\frac{B\beta_0^3}{4}\left(1 + \sqrt{1 + \frac{8(1+\xi)^2}{B^2}}\right)\right] \tag{27}$$

$$\left(B = (1 - \mathrm{e}^{-2})\Big[\frac{\pi}{3\alpha} - \frac{6\alpha}{\pi}(1+\xi)^2\Big] - [(1+\xi)^2 - 2]\mathrm{e}^{-2}\right).$$

Here $t_0 = \hat{t}_0 h^{-1}$, where $\hat{t}_0$ is defined by (8). Formula (27) is valid until the moment when the external boundary of the viscoplastic zone a reaches the cell boundary, i.e., when $a = 0.5n^{-1/3}$. We note that for a water sample with cavitational nuclei at $P^0 = -30$ MPa, $\alpha_0 = 10^{-4}$, and $R_0 = 10^{-5}$ cm, the negative-pressure relaxation time determined in [24] within the Iordanskii–Kogarko model is equal to $0.63 \cdot 10^{-8}$ sec, while according to (26) for the same conditions, it is equal to $T = 3 \cdot 10^{-8}$ sec.

Thus, a cavitating viscoelastoplastic body can be used as a generalized rheological model of cavitation development in all liquid, liquid-plastic, and solid-plastic media. In this case, the region of applicability of the generalized rheological model proposed is defined by inequality (9).

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